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SHORT COMMUNICATION

NUMERICAL SIMULATION OF STORM SURGES IN BANGLADESH USING A MULTI-LEVEL MODEL

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SUMMARY

A comparative study with a vertically integrated model and the multi-level model has shown that the former can remain a good substitute for the latter in the prediction of sea-surface elevation as long as the bottom friction coefficients in the vertically integrated models are properly specified.

KEY WORDS Storm Surge Bottom Stress Turbulent Energy Closure Exchange Coefficient Roughness Length Bottom Friction Coefficient

INTRODUCTION

In a recent paper, Dube *et al.*[1] have described a depth-averaged numerical model for the simulation of storm surges in Bangladesh. Their modelling approach was similar to that used earlier by Das *et al.*,[2,3] Johns and Ali,[4] Johns *et al.*,[5] etc. The vertically integrated equations are obtained by the omission of certain non-linear terms of advective origin in the fully three-dimensional hydrodynamic equations, together with a parameterization of the bottom stress in terms of the depth-averaged motion. The conventional use of an empirically based quadratic friction law involving the depth-averaged current raises uncertainties as to the viability of the depth-averaged models. This is because of the occurrence of a disposable friction coefficient that must be assigned a numerical value together with the fact that such a law presupposes that the bottom stress is a function of purely local flow conditions.

Recently, Johns *et al.*[6] developed a fully three-dimensional model in order to carry out an independent simulation of the storm surges along the east coast of India. Their model is based upon the turbulence energy closure scheme and is fairly sophisticated. The formulation of the frictional mechanism in this model is conceptually quite different from that used in the depth-averaged approach.

In the present paper, we have developed a multi-level model for the simulation of storm surges in Bangladesh. Our modelling approach is similar to that used by Johns *et al.*[6] Numerical experiments are performed using a wind stress forcing representative of the 1970 Chittagong cyclone and the results are compared with earlier simulations of Dube *et al.*[1] A comparison of the results indicate that a depth-averaged model is as effective as a fairly sophisticated three-dimensional model if the bottom friction is chosen judiciously.

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MODEL EQUATIONS

The sphericity of the earth's surface is neglected and we use a system of rectangular Cartesian co-ordinates in which the origin, O, is within the equilibrium level of the sea-surface. Ox points towards the south, Oy points towards the east and Oz is directed vertically upwards. The displaced position of the sea-surface is given by $z = \zeta(x, y, t)$ and the position of the sea-floor by $z = -h(x, y)$. A northern coastal boundary (the south coast of Bangladesh) is situated at $x = b_1(y)$ and there are three open-sea boundaries at $x = b_2(y)$, $y = 0$ and $y = L$ (Figure 1).

The predictive equations for the present model are the combination of those given by Johns *et al.*[6,7] The horizontal co-ordinates are transformed so as to facilitate the treatment of irregular coastal boundary and to incorporate increased resolution near the coast. This involves the introduction of a new variable, η, defined by

$$\eta = \xi + \varepsilon_0 \ln(1 + \xi/\xi_0), \tag{1}$$

where

$$\xi = \{x - b_1(y)\}/b(y), \quad b(y) = b_2(y) - b_1(y) \tag{2}$$

and ε_0 and $\xi_0(\ll 1)$ are disposable parameters.

The vertical co-ordinate is similarly transformed to facilitate the implementation of irregular bottom topography and to have increased resolution both near the sea-floor and the sea-surface. Following Johns *et al.*,[6] the new vertical variable, S, is defined by

$$\sigma + \sigma_0 = \sigma_0 \exp\{\psi(S)\}, \tag{3}$$

where

$$\sigma = (z + h)/H, \quad H = \zeta + h, \tag{4}$$

$$\psi(S) = S - \frac{1}{2S_q}(1 - \sigma_0) S^2, \tag{5}$$

$S = S_q$ at the sea-surface and $\sigma_0(\ll 1)$ is a disposable parameter. With an appropriate choice of ε_0, ξ_0 and σ_0, and with a discretization of the η and S variables having uniform grid-spacings,

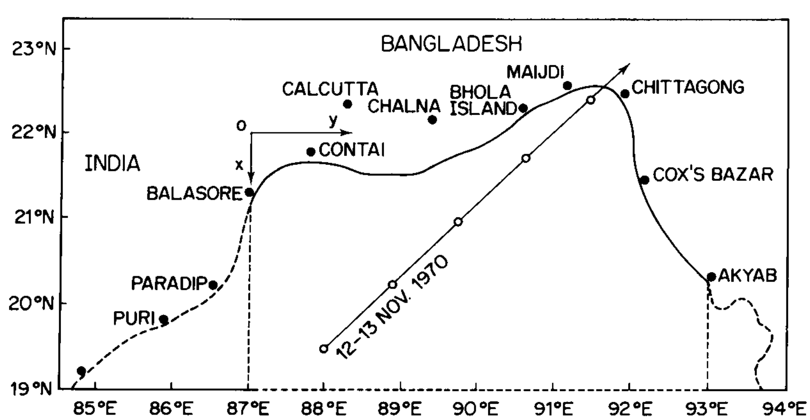

Figure 1. The analysis area and the idealized track: ------ open-sea boundaries: —O—O— 5-hourly positions of the centre of the cyclone (on the track)

there is a substantial mesh refinement near $\xi = 0$ and an increased resolution near both $\sigma = 0$ and $\sigma = 1$ compared with that in the mid-depths.

In terms of the final transformed variables, the northern and the southern lateral boundaries correspond to $\eta = 0$ and $\eta = \eta_m = 1 + \varepsilon_0 \ln(1 + 1/\xi_0)$. The sea-floor and sea-surface correspond to $S = 0$ and $S = S_q = 2\ln(1 + 1/\sigma_0)/(1 + \sigma_0)$.

Taking η, y, S and t as the new independent co-ordinates, the predictive equations are given by

$$\frac{\partial}{\partial t}(b\zeta) + \frac{1}{F}\frac{\partial}{\partial \eta}(bH\bar{U}) + \frac{\partial}{\partial y}(bH\bar{v}) = 0, \tag{6}$$

$$\frac{\partial \tilde{u}}{\partial t} + \frac{1}{F}\frac{\partial}{\partial \eta}(U\tilde{u}) + \frac{\partial}{\partial y}(v\tilde{u}) + \frac{1}{\beta}\frac{\partial}{\partial S}(\omega\tilde{u}) - f\tilde{v} = -\frac{gH}{F}\frac{\partial \zeta}{\partial \eta} + \frac{1}{H^2\beta}\frac{\partial}{\partial S}\left(\phi\frac{\partial \tilde{u}}{\partial S}\right), \tag{7}$$

$$\frac{\partial \tilde{v}}{\partial t} + \frac{1}{F}\frac{\partial}{\partial \eta}(U\tilde{v}) + \frac{\partial}{\partial y}(v\tilde{v}) + \frac{1}{\beta}\frac{\partial}{\partial S}(\omega\tilde{v}) + f\tilde{u} = -gH\left\{b\frac{\partial \zeta}{\partial y} - \left(\frac{\partial b_1}{\partial y} + \xi\frac{\partial b}{\partial y}\right)\frac{1}{F}\frac{\partial \zeta}{\partial \eta}\right\} + \frac{1}{H^2\beta}\frac{\partial}{\partial S}\left(\phi\frac{\partial \tilde{v}}{\partial S}\right), \tag{8}$$

$$\frac{\partial \tilde{E}}{\partial t} + \frac{1}{F}\frac{\partial}{\partial \eta}(U\tilde{E}) + \frac{\partial}{\partial y}(v\tilde{E}) + \frac{1}{\beta}\frac{\partial}{\partial S}(\omega\tilde{E}) = \frac{\phi}{H^3 b\beta}\left\{\left(\frac{\partial \tilde{u}}{\partial S}\right)^2 + \left(\frac{\partial \tilde{v}}{\partial S}\right)^2\right\} + \frac{1}{H^2\beta}\frac{\partial}{\partial S}\left(\phi\frac{\partial \tilde{E}}{\partial S}\right) - bH\varepsilon, \tag{9}$$

where

$(\bar{U}, \bar{u}, \bar{v})$ denote the depth-averaged values of (U, u, v),

$$(\tilde{u}, \tilde{v}, \tilde{E}) = bH(u, v, E),$$

$$U = \left\{u - \left(\frac{\partial b_1}{\partial y} + \xi\frac{\partial b}{\partial y}\right)v\right\}\Big/ b(y),$$

$$\omega = \frac{\partial \sigma}{\partial t} + u\frac{\partial \sigma}{\partial x} + v\frac{\partial \sigma}{\partial y} + w\frac{\partial \sigma}{\partial z}$$

$$F(\eta) = \frac{\partial \xi}{\partial \eta} = 1/\{1 + \varepsilon_0/(\xi + \xi_0)\}$$

$$\beta(S) = \frac{\partial \sigma}{\partial S}, \quad \phi = \frac{K}{\beta},$$

$K = C^{1/4} l E^{1/2}$ is the exchange coefficient, $\varepsilon = C^{3/4} E^{3/2} l^{-1}$ is the dissipation term, $C = 0{\cdot}08$ is an empirical constant and l is the length scale of the vertical mixing process determined from

$$l = k\left\{(E/E_0)^{1/2} z_0 + HE^{1/2}\int_0^S E^{-1/2}\beta(S)\,dS\right\}, \tag{10}$$

subject to $l = kz_0$ at $S = S_q$, k being the von Karman's constant $(= 0{\cdot}4)$ and z_0 the roughness length.

The applied surface wind stress conditions are

$$\left(\frac{\partial \tilde{u}}{\partial S}, \frac{\partial \tilde{v}}{\partial S}\right) = \frac{bH^2}{\rho\phi}(\tau_x^\zeta, \tau_y^\zeta) \qquad \text{at } S = S_q, \tag{11}$$

and the appropriate boundary conditions are given by[6,7]

$$U = 0, \qquad \text{at } \eta = 0 \tag{12}$$

$$b(y)\bar{U} - \left(\frac{g}{h}\right)^{1/2} \zeta = 0, \qquad \text{at } \eta = \eta_m, \tag{13}$$

$$\bar{v} + \left(\frac{g}{h}\right)^{1/2} \zeta = 0, \qquad \text{at } y = 0, \tag{14}$$

$$\bar{v} - \left(\frac{g}{h}\right)^{1/2} \zeta = 0, \qquad \text{at } y = L, \tag{15}$$

$$u = v = \omega = 0, \quad \text{at} \quad S = 0; \quad \omega = 0, \quad \text{at} \quad S = S_q \tag{16}$$

and

$$\frac{\partial \tilde{E}}{\partial S} = 0, \quad \text{at} \quad S = 0 \quad \text{and} \quad S = S_q. \tag{17}$$

The equations (6)–(9) are solved numerically by using a horizontal staggered grid of the type described by Johns *et al.*[6,7]

NUMERICAL EXPERIMENTS

Numerical experiments are performed by using an analysis area which extends from 87 °E to 93 °E along the south coast of Bangladesh and there are three open-sea boundaries situated along 87 °E, 93 °E, and 19 °N (Figure 1). Thus, on an average, the width of the coastal zone is about 270 km and the east–west extent, L, of the analysis area is about 600 km. An idealized cyclone moves along the indicated straight line track shown in Figure 1. The wind field associated with this cyclone is simulated by applying the following empirically-based formula:[8]

$$V = \begin{cases} V_0\left(\dfrac{r}{R}\right)^{3/2}, & \text{for } r \leqslant R, \\ V_0\left(\dfrac{R}{r}\right)^{1/2}, & \text{for } r > R, \end{cases} \tag{18}$$

where V_0 is the maximum sustained wind, R is the radius of maximum wind and r is the distance from the centre of the cyclone. Following Das *et al.*[2] we take $V_0 = 50\,\text{ms}^{-1}$ and $R = 40\,\text{km}$.

The governing equations (6)–(9) form the basis for a set of finite difference analogues. We write

$$\eta = \eta_i = (i-1)\Delta\eta, \quad i = 1(1)m, \quad \Delta\eta = \eta_m/(m-1),$$
$$y = y_j = (j-1)\Delta y, \quad j = 1(1)n, \quad \Delta y = L/(n-1),$$
$$S = S_k = (k-1)\Delta S, \quad k = 1(1)q, \quad \Delta S = S_q/(q-1),$$

where m and n are, even and odd integers, respectively.

In this study we have taken $m = 10$, $n = 21$, $S = 11$, $\xi_0 = 0{\cdot}001$, $\varepsilon_0 = 0{\cdot}04$ (implying that $\eta_m \simeq 1{\cdot}27$ and $\Delta\eta \simeq 0{\cdot}14$), $\sigma_0 = 0{\cdot}0001$ and $z_0 = 2\,\text{mm}$. This prescription implies that the first offshore grid point at which the elevation is computed is, on average, about 5 km from the coastline and $\Delta y \simeq 30$ km. The adequacy of this grid resolution has already been tested by Dube *et al.*[1]

The vertical resolution is of paramount importance and our setting implies that the first computational level above the sea-floor is about 2 mm near the coastline and 20 cm in the region of deepest water. The corresponding values of the grid-spacing immediately below the sea-surface are about 30 cm and 36 m.

In order to maintain computational stability, the time step, Δt, is taken to be 3 min.

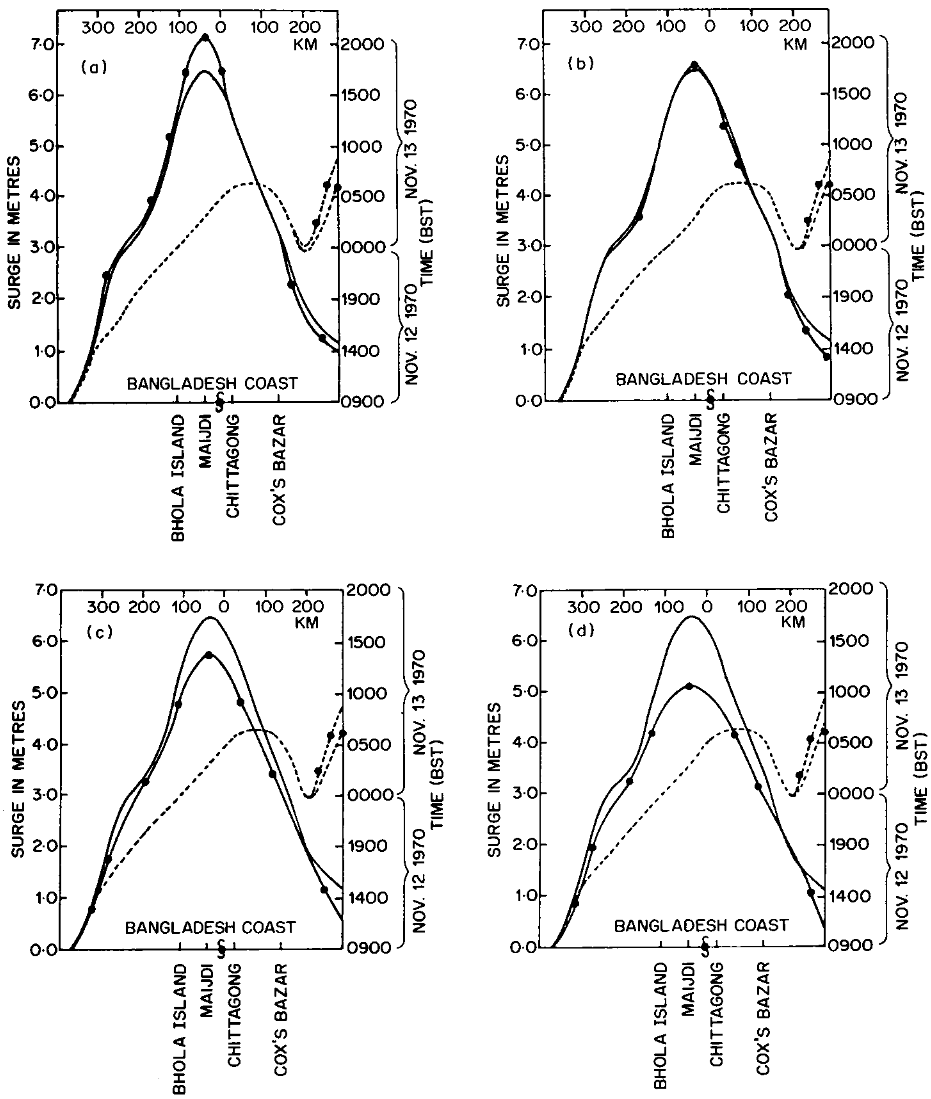

Figure 2. Maximum predicted sea-surface elevation and its time of occurrence along the Bangladesh coast: § place of landfall; ● time of landfall (on the time axis);———peak surge envelope (MLM);-----time of occurrence (MLM);—●—●—peak surge envelope (DAM) with (a) $c_f = 1{\cdot}3 \times 10^{-3}$, (b) $c_f = 2{\cdot}6 \times 10^{-3}$, (c) $c_f = 5{\cdot}2 \times 10^{-3}$ and (d) $c_f = 7{\cdot}8 \times 10^{-3}$;--●--● time of occurrence (DAM)

RESULTS AND DISCUSSION

Our principal objective is to compare the results of the multi-level model (MLM) with our earlier depth-averaged model (DAM), each of these being characterized by a totally different representation of the frictional mechanism. A further question that remains to be answered is the proper choice of the bottom friction coefficient, c_f to be used in the DAM so that it produces surge elevations and phases in the same range as those obtained from MLM. We have investigated this by using DAM with $c_f = 1{\cdot}3 \times 10^{-3}, 2{\cdot}6 \times 10^{-3}, 5{\cdot}2 \times 10^{-3}, 7{\cdot}8 \times 10^{-3}$ and give the corresponding peak surge envelope and its time of occurrence in Figures 2(a), 2(b), 2(c), 2(d), respectively. It may be seen from the Figure that the bottom friction coefficient $c_f = 2{\cdot}6 \times 10^{-3}$ gives the best agreement between the results from the two models. We note that higher values of c_f lead to lower surge responses. At Maijdi, the lowest value of $c_f (= 1{\cdot}3 \times 10^{-3})$ produces a peak surge about 28 per cent higher than that obtained with $c_f = 7{\cdot}8 \times 10^{-3}$. Thus, the elevations are clearly sensitive to the bottom friction coefficient when using the depth-averaged model and $2{\cdot}6 \times 10^{-3}$ is the best choice according to results from the multi-level model.

Figure 3 depicts the time variation of the sea-surface elevation at the coastal stations Bhola Island, Maijdi, Chittagong and Cox's Bazar. The responses have been compared with the corresponding sea-surface elevations determined from DAM using a uniform bottom friction coefficient $c_f = 2{\cdot}6 \times 10^{-3}$. It may be seen from the Figure that the surge response computed from the two models is both qualitatively and quantitatively similar. After the peak surge, we note that both MLM and DAM predict a rapidly falling sea-surface elevation with DAM producing a greater peak negative surge than MLM by about 8 per cent and 16 per cent at Bhola Island and

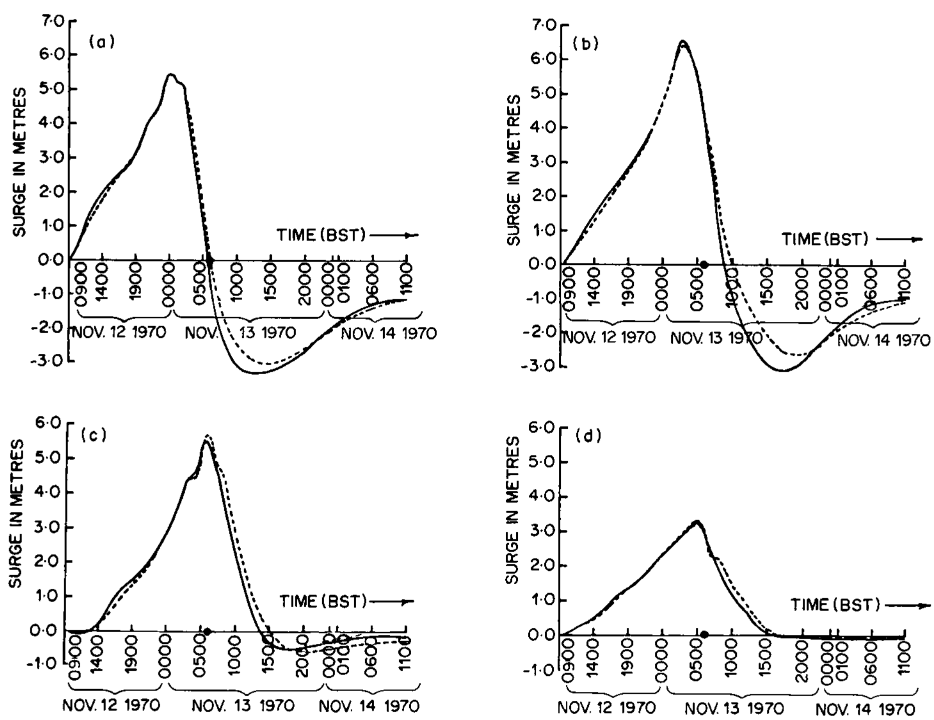

Figure 3. Time variation of the predicted sea-surface elevation at (a) Bhola Island, (b) Maijdi, (c) Chittagong and (d) Cox's Bazar: ● time of landfall; ——— MLM; ----- DAM with $c_f = 2{\cdot}6 \times 10^{-3}$

Maijdi, respectively. We also note that the computed phase differences between MLM and DAM are insignificant and that the peak elevations in the two models therefore occur at the same time.

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